Group # ____ Names:

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Calculus I ______________________________

Lab Project: The Relationship between a Function and its Derivative

Objective:

To understand the relationship between the graph of a function and the graph of its derivative[1]

Analysis

1. Let $f(x) = x(x+1)^2(x-1)(x-2)$. Plot the graphs of f and f' in the viewing rectangle defined by $-1.2 \leq x \leq 2.2$ and $-5.2 \leq y \leq 4$.

(a) What are the x-coordinates of the local extrema of f?

(b) What are the zeros of f'?

(c) Over what intervals does the graph of f appear to be increasing?

(d) Over what intervals does the graph of f' appear to be above the x-axis?

(e) Over what intervals does the graph of f appear to be decreasing?

(f) Over what intervals does the graph of f' appear to be below the x-axis?

[1]Portions of this lab have been adapted with permission from the *Resources for Calculus Collection* (copyright 1993) published by the MATHEMATICAL ASSOCIATION OF AMERICA.

2. Let $f(x) = \dfrac{x}{x^2+1}$. Plot the graphs of f and f' in the viewing rectangle defined by $-4 \le x \le 4$ and $-1 \le y \le 1$.

(a) What are the x-coordinates of the local extrema of f ?

(b) What are the zeros of f' ?

(c) Over what intervals does the graph of f appear to be increasing?

(d) Over what intervals does the graph of f' appear to be above the x-axis?

(e) Over what intervals does the graph of f appear to be decreasing?

(f) Over what intervals does the graph of f' appear to be below the x-axis?

3. On the basis of your experience thus far, complete the following statements:

(a) If the graph of f' is above the x-axis on an interval I, then f is ____________________

(b) If the graph of f' is below the x-axis on an interval I, then f is ____________________

(c) If f has a local extreme value at $x = a$, then $f'(a)$ ____________________

4. Let $f(x) = |x^2 - x - 2|$. Plot the graphs of f and f' in the viewing rectangle defined by $-2 \le x \le 3$ and $-5 \le y \le 5$.

(a) What are the x-coordinates of the local extrema of f ?

(b) What are the zeros of f' ? Where is f' undefined?

5. Did your response to Exercise 3(c) address the phenomenon observed in Exercise 4? If not, then modify your response.

6. The graph in Figure 1 is a sketch of the <u>derivative</u> of a function f.

(a) Construct a *qualitatively correct* graph of f''. Justify your answer.

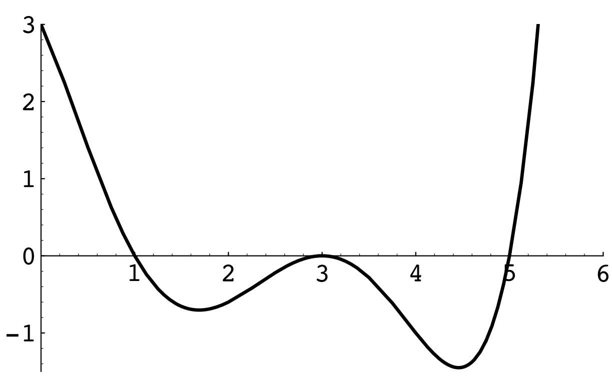

Figure 1: $y = f'(x)$

Sketch $y = f''(x)$ here.

6.(b) The graph in Figure 2 is identical to that in Figure 1. Using this graph, construct a *qualitatively correct* graph of f. Justify your answer. Explain why there isn't a unique function that has f' as its derivative.

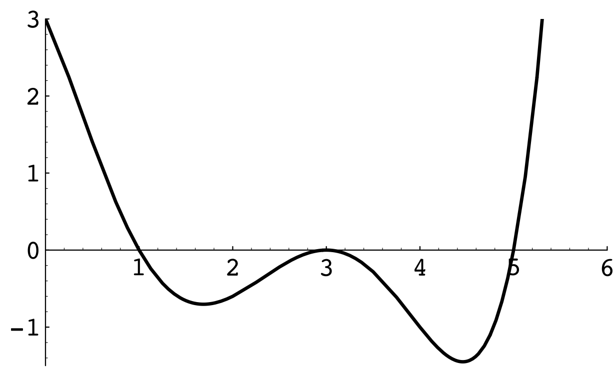

Figure 2: $y = f'(x)$

Sketch a plausible graph of $y = f(x)$ here.

7. Compare the graphs of f and f'' in Exercise 6. Describe how the *concavity* f is related to properties of its second derivative f''.