Rules of Differentiation

Suppose f and g are differentiable on an open interval I. Then for all $x \in I$ we have the following:

Constant Multiple Rule. $\frac{d}{dx}[k \cdot f(x)] = k \cdot f'(x)$, *where k is a constant.*

Sum Rule. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

Product Rule. $\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) \ + \ g(x) \cdot f'(x)$

Quotient Rule. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) \ - \ f(x) \cdot g'(x)}{[g(x)]^2}$, *where $g(x) \neq 0$.*

Chain Rule. *If h is differentiable on an interval J and $g(x) \in J$ for each $x \in I$, then*

$$\frac{d}{dx}[(h \circ g)(x)] = h'(g(x)) \cdot g'(x)$$

Derivatives of Selected Elementary Functions

Each identity below holds for every x at which all of the expressions involved are defined. The parameters b, C, and k represent arbitrary constants.

(1) Power Rule. $\frac{d}{dx}(x^k) = k \cdot x^{k-1}$

(2) Exponentials. $\frac{d}{dx}(b^x) = b^x \cdot \ln b \quad (b > 0)$ *and* $\frac{d}{dx}(e^{kx}) = k \cdot e^{kx}$

(3) Logarithms. $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$ *and* $\frac{d}{dx}(\log_b|x|) = \frac{1}{x \cdot \ln b} \quad (b > 0 \text{ and } b \neq 1)$

(4) Trig Functions. $\frac{d}{dx}(\sin x) = \cos x$, $\frac{d}{dx}(\cos x) = -\sin x$, $\frac{d}{dx}(\tan x) = \sec^2 x$,
$\frac{d}{dx}(\cot x) = -\csc^2 x$, $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$, $\frac{d}{dx}(\csc x) = -\csc x \cdot \cot x$

(5) Inverse Trig Functions. $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$,
$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$, $\frac{d}{dx}(\text{arcsec}\, x) = \frac{1}{x\sqrt{x^2-1}}$

Antiderivatives of Selected Elementary Functions

(I) Power Rule. *If* $f(x) = x^k \quad (k \neq -1)$ *and* $F(x) = \frac{x^{k+1}}{k+1} + C$ *then* $F' = f$.
If $g(x) = x^{-1}$ *and* $G(x) = \ln|x| + C$ *then* $G' = g$.

(II) Exponentials. *If* $f(x) = e^{kx} \quad (k \neq 0)$ *and* $F(x) = \frac{e^{kx}}{k} + C$ *then* $F' = f$.
If $g(x) = b^x \quad (b > 0, b \neq 1)$ *and* $G(x) = \frac{b^x}{\ln b} + C$ *then* $G' = g$.

(III) Logarithms. *If* $f(x) = \ln x$ *and* $F(x) = x \ln x - x + C$ *then* $F' = f$.

(IV) Trig Functions. *If* $f(x) = \sin x$ *and* $F(x) = -\cos x + C$ *then* $F' = f$.
If $g(x) = \cos x$ *and* $G(x) = \sin x + C$ *then* $G' = g$.