Calculus I Exam 1

theme for today's test: do the robot.

Name: KEY

Pledge:

True/False. (2 pts each)

$(x-4)(x+1)$ so $x \neq 4, -1$

T (F) 1) $f(x) = \dfrac{\cos x}{x^2 - 3x - 4} + \dfrac{1}{\sqrt{x}}$ is continuous only on the interval $(0,3) \cup (3,4) \cup (4,\infty)$.

$(0, \infty)$ $(0,4) \cup (4, \infty)$

T (F) 2) $\sin 2x = 2 \sin x$

(T) F 3) $f(x-5)$ shifts the function $f(x)$ to the right by 5.

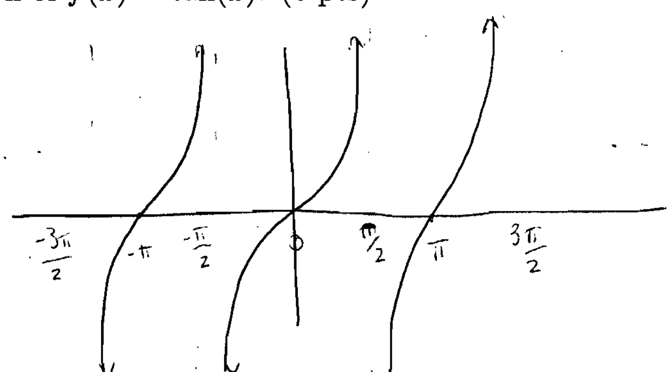
(T) F 4) I know how to do the robot. On this test. And on the dance floor.

T (F) 5) $\lim_{x \to 0^+} \ln x = \infty$.

T (F) 6) If $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$, then f is continuous at c.

Problems, conceptual and otherwise.

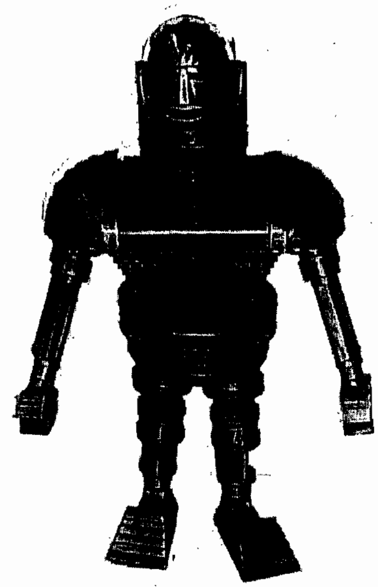
1) Draw the graph of $f(x) = \tan(x)$. (6 pts)



2) Simplify the following (6 pts): $\log_2 6 + 2\log_2 2 - \log_2 3$

$\log_2 6 + \log_2 2^2 - \log_2 3$

$\log_2 \left(\frac{6 \cdot 4}{3}\right) = \log_2(8) = \boxed{3}$



Find the following limits. Show all work. (8 pts each)

3a) $\displaystyle\lim_{x\to 3^-} \frac{x+2}{x^2-2x-3}$

$$\lim_{x\to 3^-} \frac{\overset{\to 5}{(x+2)}}{\underbrace{(x-3)}_{\text{neg small \#}}\underbrace{(x+1)}_{\to 4}} = -\infty$$

3b) $\displaystyle\lim_{x\to 0} \frac{\tan x}{\sin 3x}$

$$\lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{\sin 3x} = \lim_{x\to 0} \frac{\sin x}{\cos x}\left(\frac{1}{\sin 3x}\right) = \lim_{x\to 0} \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{1}\right)\left(\frac{x}{x}\right)\left(\frac{1}{\sin 3x}\right)\left(\frac{3x}{3x}\right)$$

$$= \lim_{x\to 0} \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{x}\right)\left(\frac{3x}{\sin 3x}\right)\left(\frac{\cancel{x}}{3\cancel{x}}\right) = 1\cdot 1\cdot 1\cdot \frac{1}{3} = \boxed{\frac{1}{3}}$$

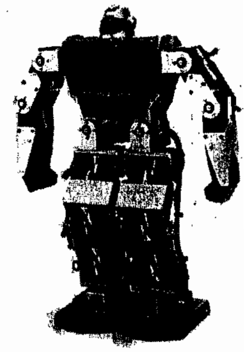
3c) Use squeeze theorem to show that $\displaystyle\lim_{x\to 0} \sqrt{x}\sin(1/x) = 0$.

$$-1 \le \sin\left(\frac{1}{x}\right) \le 1$$

$$-\sqrt{x} \le \sin\left(\frac{1}{x}\right) \le \sqrt{x}$$

$$\lim_{x\to 0} -\sqrt{x} = 0 \qquad \text{so by Squeeze Thm,}$$

$$\lim_{x\to 0} \sqrt{x} = 0 \qquad \lim_{x\to 0} \sqrt{x}\sin\left(\frac{1}{x}\right) = 0.$$



4) What is the mathematical definition of continuity? (4 pts)

f is cont. at c if $\lim_{x \to c} f(x) = f(c)$.

5) Mathematically determine a constant c that make the following function continuous. (8 pts)

$$f(x) = \begin{cases} 3c - x & \text{if } x \leq 1, \\ c^2 + cx & \text{if } x > 1. \end{cases}$$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 3c - x = 3c - 1.$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} c^2 + cx = c^2 + c.$

Also, at $x = 1$, $f(1) = 3c - 1$.

If $3c - 1 = c^2 + c$, f will be continuous.

$\to 0 = c^2 + c - 3c + 1$

$0 = c^2 - 2c + 1$

$0 = (c-1)(c-1)$

at $c = 1$,

f will be cont.

6) Determine where the following function is discontinuous and the type of discontinuity (8 pts):

$f(x) = \left(\frac{2x}{|x-4|} + \frac{8}{x-4} \right)$ (8 pts)

at $x = 4$.

$\lim_{x \to 4^-} \frac{2x}{|x-4|} + \frac{8}{x-4} = \lim_{x \to 4^-} \frac{2x}{-(x-4)} + \frac{8}{x-4} = \lim_{x \to 4^-} \frac{-2x+8}{x-4}$

$= \lim_{x \to 4^-} \frac{-2\cancel{(x-4)}}{\cancel{x-4}} = -2$

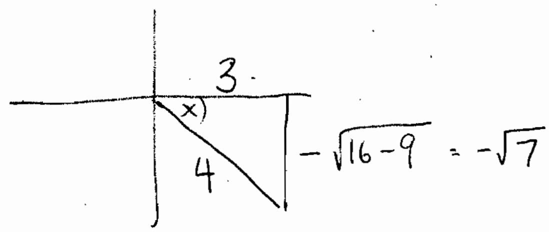
$\lim_{x \to 4^+} \frac{2x}{|x-4|} + \frac{8}{x-4} = \lim_{x \to 4^+} \frac{2x+8}{x-4} = \lim_{x \to 4^+} \frac{2(x+4)}{x-4} = \infty$

8

small pos #

So ∞ discont at $x = 4$.

7a) Let $\sec x = 4/3$, where $x \in [3\pi/2, 2\pi]$. Find $\sin 2x$. (6 pts)



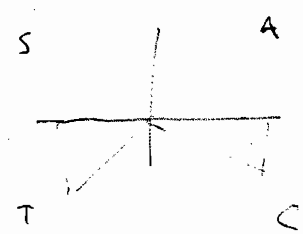
$$\sin 2x = 2 \sin x \cos x = 2\left(\frac{-\sqrt{7}}{4}\right)\left(\frac{3}{4}\right)$$

$$= \boxed{-\frac{6\sqrt{7}}{16}}$$

7b) Solve for x in the interval $[0, 2\pi)$ (6 pts): $2\sin x + 1 = 0$

$$2 \sin x = -1$$

$$\sin x = -1/2$$



$$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$$

7c) (4 pts) $\cos^{-1}(\sqrt{3}/2) =$

$$\cos^{-1}(\sqrt{3}/2) = \theta \Leftrightarrow \cos\theta = \frac{\sqrt{3}}{2}$$

$$\boxed{\theta = \pi/6}$$

8) Find a domain where the following function is one-to-one and find its inverse on that domain (8 pts): $f(x) = \dfrac{1}{3x-8}$

$x \neq \frac{8}{3}$, $\boxed{D: \mathbb{R} \setminus \{\frac{8}{3}\}}$

$y = \dfrac{1}{3x-8}$

$(3x-8)y = 1$

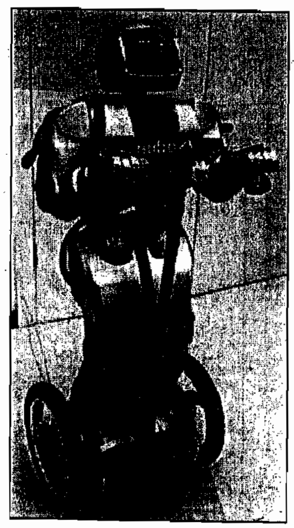
$3xy - 8y = 1$

$\dfrac{3xy}{3y} = \dfrac{1+8y}{3y}$

$x = \dfrac{1+8y}{3y}$

$\boxed{f^{-1}(x) = \dfrac{1+8x}{3x}}$

9) A robot on a segway (see picture below) zooms along a path given by the equation $f(t) = t^2$, with units in feet and minutes. What is its instantaneous velocity after 1 minute? Absolutely NO credit will be given if you just take the derivative. (8 pts)



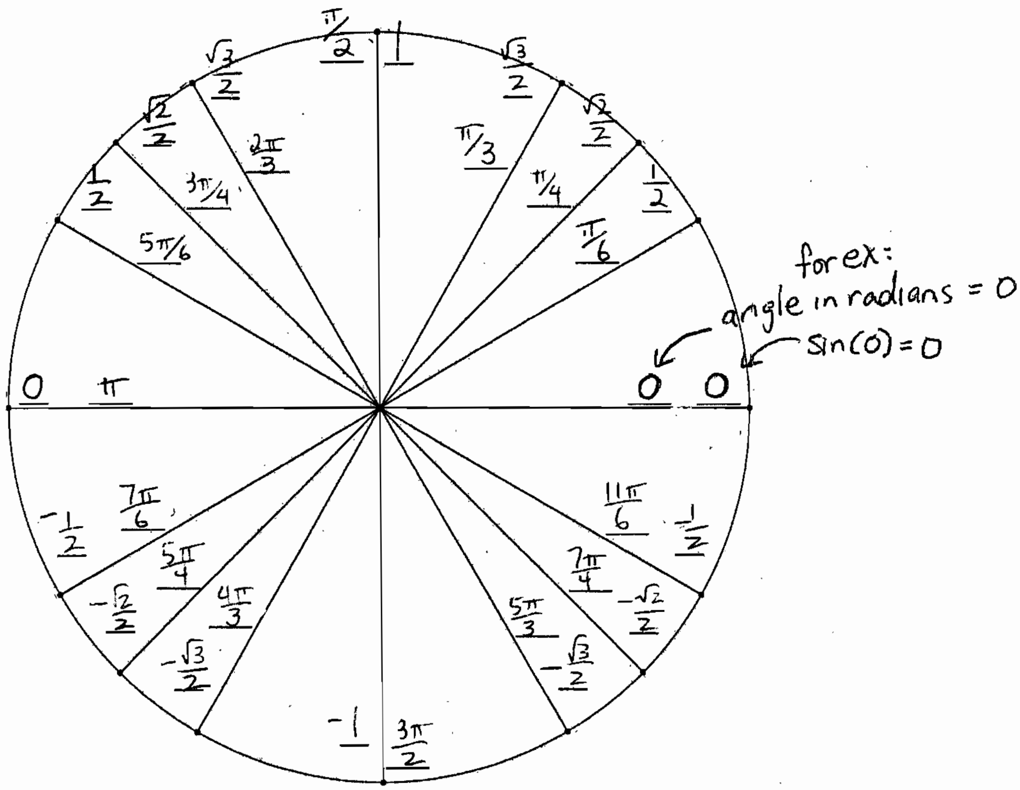
$\lim_{t \to 1} \dfrac{t^2 - 1^2}{t-1} = \lim_{t \to 1} \dfrac{\cancel{(t-1)}(t+1)}{\cancel{(t-1)}} = 2$

$\boxed{2 \text{ ft/min}}$

EXTRA EXTRA EXTRA EXTRA.

extra credit (point value to be determined):

1. Fill in the following the angle and the value for $\sin x$:



$$\sin 2x = 2 \sin x \cos x$$

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$