KEY

Calculus I Derivatives Worksheet

Find the derivative of $f(x) = \frac{1}{x+1}$ using the limit definition.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{(x+h)+1} - \frac{1}{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x+1-(x+h+1)}{(x+h+1)(x+1)}}{h} = \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(x+h+1)(x+1)}$$

$$= \boxed{\frac{-1}{(x+1)(x+1)}}$$

Find the derivatives of the following functions using the limit laws (not the limit definition):

a) $f(x) = 5x^4$

$$20x^3$$

b) $f(x) = x^{2/5} + x^4$

$$\frac{2}{5}x^{-3/5} + 4x^3$$

c) $f(x) = x^8 3^x$

$$x^8(\ln 3)3^x + 8x^7 3^x$$

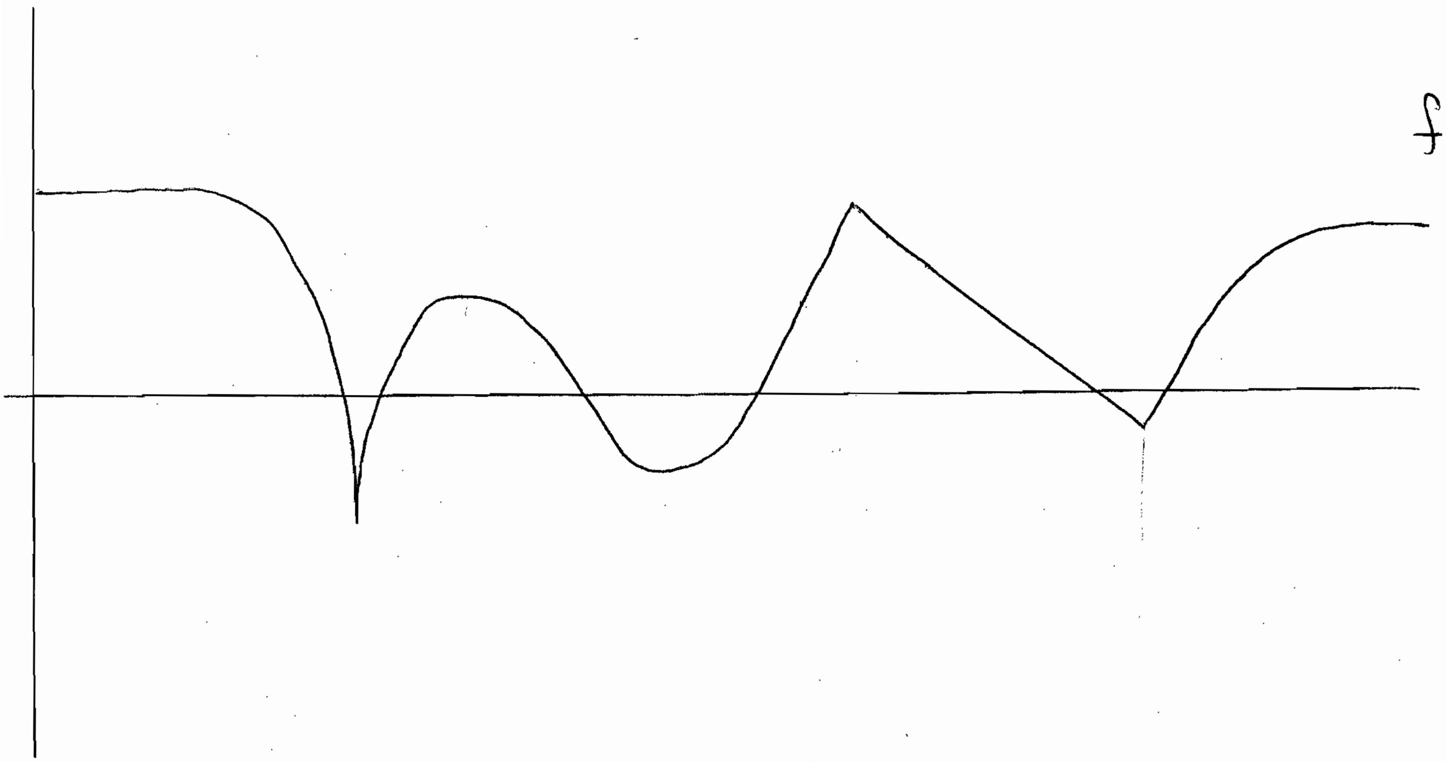
d) $f(x) = \frac{x}{\frac{1}{x}+1} = \frac{x}{x^{-1}+1}$

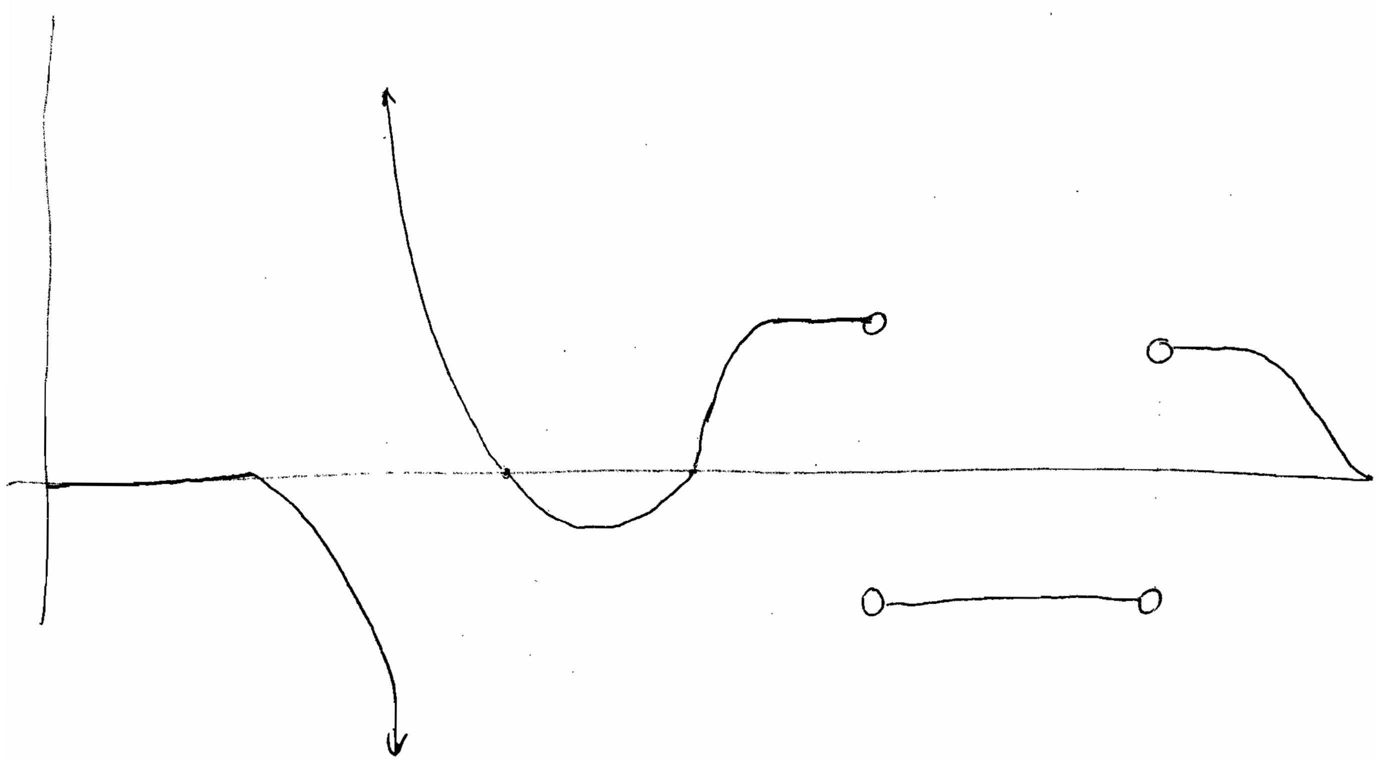
$$\frac{(x^{-1}+1)(1) - (x)(-1x^{-2})}{(x^{-1}+1)^2}$$

e) $f(x) = (x)(x^2 + 5x)(e^x)$

$$x\left((x^2+5x)e^x + (2x+5)e^x\right) + (1)((x^2+5x)(e^x))$$

Draw the graph of f' given the graph of f below.





2) Assume f and g are differentiable functions and we have the following table of data:

x	f(x)	f'(x)	g(x)	g'(x)
-2	3	1	-5	8
-1	-9	7	4	1
0	5	9	9	-3
1	3	-3	2	6
2	-5	3	8	?

2a) Let $h(x) = x^{4/3} f(x)$. What is $h'(-1)$?

$$h'(x) = \frac{4}{3}x^{1/3} f(x) + x^{4/3} f'(x)$$

$$h'(-1) = \frac{4}{3}(-1)^{1/3}(-9) + (-1)^{4/3}(7) = 12 + 7 = 19$$

2b) Let $j(x) = -4 f(x) g(x)$. What is $j'(1)$?

$$j'(x) = -4(f'(x)g(x) + f(x)g'(x))$$

$$j'(1) = -4((-3)(2) + (3)(6)) = -48$$

2c) Let $k(x) = (x f(x))/(g(x))$ What is $k'(-2)$?

$$k'(x) = \frac{g(x)(f(x) + xf'(x)) - xf(x)(g'(x))}{g(x)^2}$$

$$k'(-2) = \frac{(-5)(3 + (-2)(1)) - (-2)(3)(8)}{(-5)^2} = \frac{-5+48}{25} = \frac{43}{25}$$

2d) Let $l(x) = x^3 g(x)$. If $l'(2) = -48$, what is $g'(2)$?

$$l'(x) = 3x^2 g(x) + x^3 g'(x)$$

$$-48 = 3(2)^2(8) + 2^3 g'(x)$$

$$-48 = 96 + 8g'(x)$$

$$-144 = 8g'(x)$$

$$g'(x) = \frac{-144}{8} = -18$$

3) Find the equations of the tangent lines to $y = (x-1) / (x+1)$ parallel to the line $x-2y = 2$.

$$y' = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$$

$$y' = \frac{x+1 - x+1}{(x+1)^2} = \frac{2}{(x+1)^2} = \text{slope of tangent line.}$$

$$x - 2y = 2 \Rightarrow \frac{-2y}{-2} = \frac{-x}{-2} + \frac{2}{-2} \Rightarrow y = \frac{1}{2}x - 1$$

parallel means slope $= \frac{1}{2}$

$$\frac{2}{(x+1)^2} = \frac{1}{2} \qquad (x+1)^2 = 4$$

$$\pm(x+1) = 4 \Rightarrow x+1 = 4 \Rightarrow x = 3$$

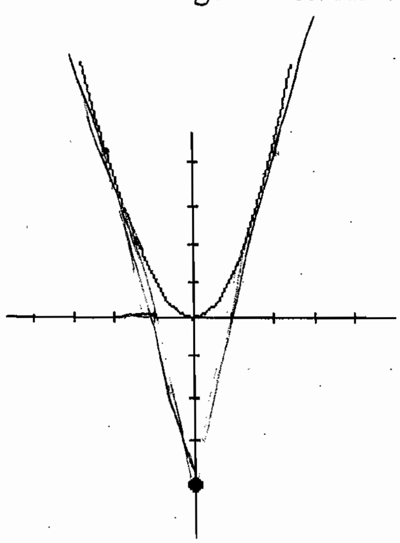
$$-x-1 = 4 \Rightarrow x = -5$$

2 pts: $(3, \frac{2}{4}), (-5, \frac{-6}{-4})$

2 equations: $\frac{1}{2} = \frac{y - \frac{1}{2}}{x-3} \Rightarrow y = \frac{1}{2}x - 1$

$\frac{1}{2} = \frac{y - 3/2}{x+5} \Rightarrow y = \frac{1}{2}x + 1$

4) There are two tangent lines to the parabola $y = x^2$ that go through the point $(0, -4)$. Sketch these tangent lines. At which points do these tangent lines touch the curve?



$y' = 2x$ = slope of tangent line.

2 pts: $(0, -4)$, (x, x^2).
on tangent line

$$\text{So } 2x = \frac{x^2 - (-4)}{x - 0}$$

$$2x^2 = x^2 + 4$$

$$x^2 = 4$$

$$\boxed{x = \pm 2}$$

at $\boxed{(2, 4) \text{ and } (-2, 4)}$