# Calculus I Continuity Worksheet

1. Determine the discontinuities of the following functions and state the type of discontinuity.

a. $f(x) = \dfrac{x^2+3x}{x+3}$

rational function, Cont. on its domain : $\mathbb{R} \setminus \{-3\}$

$f(-3)$ is not defined.

$$\lim_{x\to -3^-} \frac{x^2+3x}{x+3} = \lim_{x\to -3^-} \frac{x\cancel{(x+3)}}{\cancel{x+3}} = \lim_{x\to -3^-} x = -3$$

$$\lim_{x\to -3^+} \frac{x^2+3x}{x+3} = \lim_{x\to -3^+} x = -3$$

So removable discont at $x=-3$

b. $f(x) = \dfrac{3}{x-5}$

Cont on its domain : $\mathbb{R} \setminus \{5\}$

$$\lim_{x\to 5^-} \frac{3}{x-5} = -\infty$$

$$\lim_{x\to 5^+} \frac{3}{x-5} = \infty$$

(b/c by plugging in 4.9, 4.99, etc., the denominator is becoming a smaller and smaller negative #, so $\frac{3}{x-5}$ goes to $-\infty$.)

So $\boxed{\infty \text{ discont.}}$ at $x=5$

c. $f(x) = \begin{cases} x & \text{if } x < 2, \\ x^2 & \text{if } x \geq 2, \end{cases}$

b/c we are looking at x values to the left of 2

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x = 2$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^2 = 4$$

$\boxed{\text{Jump discont.}}$ at $x=2$

cont. for $x<2$ and $x>2$ since the polynomials $x$ and $x^2$ are cont

d. $f(x) = \dfrac{\sin x}{x}$

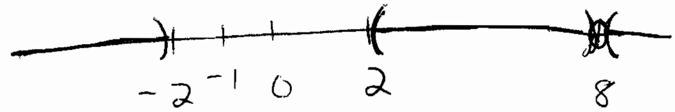
From class, we know $\displaystyle\lim_{x\to 0} \frac{\sin x}{x} = 1$ (approaching from both sides)

so since $\frac{\sin x}{x}$ is not defined at 0 ($f(0)$ DNE),

there is a $\boxed{\text{removable discont}}$ at $x=0$.

Cont. everywhere else : combination of trig and polynomial.

2. Given $f(x) = \dfrac{x^2-6x-16}{(x^2-7x-8)\sqrt{x^2-4}}$, determine the intervals where $f$ is continuous.




Continuous on its domain.

$$\frac{x^2-6x-16}{(x-8)(x+1)\sqrt{(x-2)(x+2)}} \leftarrow x^2-4>0$$

$x \neq 8$ $\quad$ $x \neq -1$

$x^2 > 4$

$\pm x > 2 \Rightarrow x > 2,\ x < -2$

$$\boxed{(-\infty,-2) \cup (2,8) \cup (8,\infty)}$$

3. Determine whether the following piecewise defined function is continuous at $c$. Verify your answer, and state the type of dicontinuity.

$$f(x) = \begin{cases} \dfrac{x^2+2}{x+1} & \text{if } x < 0, \\[2ex] \dfrac{-4}{x-2} & \text{if } 0 < x \leq 3, \\[2ex] x^2-3 & \text{if } x > 3. \end{cases}$$

$C = -1, 0, 2, 3$

as $x \to -1$ from the left, $x^2+2$ gets close to 3

$\underline{C=-1}$: $\displaystyle\lim_{x\to -1^-} \frac{x^2+2}{x+1} = -\infty$ $\quad$ $x+1$ becomes a small neg. #

$\boxed{\infty \text{ discont}}$ at $-1$

$\underline{C=0}$: $\displaystyle\lim_{x\to 0^-} \frac{x^2+2}{x+1} \overset{DSP}{=} 2$

$\displaystyle\lim_{x\to 0^+} \frac{-4}{x-2} \overset{DSP}{=} 2$

$\boxed{\text{removable}}$ b/c $f(0)$ is undefined

$\underline{C=2}$: $\displaystyle\lim_{x\to 2^-} \frac{-4}{x-2} = \infty$ $\quad$ $x-2 \to$ small neg #

$\boxed{\infty \text{ discont}}$ at $C=2$

$\underline{C=3}$: $\displaystyle\lim_{x\to 3^-} \frac{-4}{x-2} = \frac{-4}{1} = -4$

$\displaystyle\lim_{x\to 3^+} x^2-3 = 9-3 = 6$

$\boxed{\text{jump discont}}$ at $c=3$